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ANSTICE
FALLING BODIES
1794

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AN
ENQUIRY
INTO THE
LAWS
OF
FALLING BODIES, &c.

BY *ROBERT ANSTICE.*

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To the Preſident, Council, and Fellows of the Royal Society of London, for the improving of Natural Knowledge.

GENTLEMEN,

IF the following Treatiſe may anſwer the Purpoſe of extending uſeful Knowledge, or clearing up any doubt, as to the true operation of Mechanical Powers, the Approbation of your learned Body will be its beſt recommendation to the peruſal of thoſe for whoſe Uſe it is intended; it is therefore reſpectfully ſubmitted to your determination by

Your Obedient,

Humble Servant,

THE AUTHOR.

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INDEPENDENTLY of the Satisfaction ariſing from the inveſtigation of abſtract Truth, the utility of duly underſtanding whatever relates to the Subject of the following Pages, muſt be acknowledged by all who conſider the general application of Machinery to almoſt every branch of our Manufactories, to be both expedient and neceſſary; expedient, as much cheaper than manual labour, and neceſſary, as manual labour is inadequate in many caſes to the Fabrication, and in others to the demand of our Manufactures.

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The very reſpectable authorities to which the Author found he had to oppoſe his Opinions in ſeveral parts of this Enquiry, for ſome time prevented his publiſhing them; but having been ſucceſsful in the application of thoſe of his Principles which he has reduced to Practice, and his Treatiſe on Wheel Carriages having met with a favourable reception, he is induced to lay them before the Public.

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AN ENQUIRY, &c.

SECTION. I.

Of the Deſcent of Heavy Bodies near the Surface of the Earth.

EXPERIMENTS have diſcovered, that a heavy body, near the ſurface of the earth, will fall in one ſecond of time through 16 ft. 2 in. of void ſpace, and that this velocity conſtantly accelerates in the ratio of the ſquares of the times in which the body falls: from which we may determine, that in two ſeconds it will be carried through a fall of 64 ft. 8 in. or 4 (ſquare of 2) times 16 ft. 2 in. in 3 ſeconds through 145 ft. 6 in. which is 9 (ſquare of 3) times 16 ft. 2 in. and ſo on, increaſing as the odd numbers 1, 3, 5, 7, 9, &c. For if it fell through one given ſpace in a certain time, in the next equal time it would fall through three ſuch ſpaces; which added to 1 is 4, (the ſquare of 2); in the 3d equal time it would go through 5, which added to what it went before, is 9 (the ſquare of 3) and ſo on.—And the reaſon of this proportion in the acceleration will be evident, if we conſider that a body firſt yielding to the action of gravity, begins to fall with a motion, the ſmalleſt poſſible; but by the continuation of that action upon it, it's motion is conſtantly and regularly increaſed, till we ſuppoſe it arrived to a certain place in a correſpondent time, both which ſpace and time we will denote by the number 1.—Now as it began it's fall (from a ſtate of reſt,) with the leaſt motion poſſible, which was uniformly increaſed by the action of gravity, a medium of the degree of motion it had in falling would carry it equably through the ſame ſpace, in the ſame time, if the cauſe of the firſt motion ceaſed to act on, and accelerate it;—the motion therefore it has acquired at it's arrival at this place muſt be double the *medium* of that acting on it in it's fall; conſequently, in the ſame time would carry it through double the ſpace, without the cauſe continuing to act on it; but as that cauſe, when it does act, is ſufficient to carry it through the ſame ſpace in the ſecond given time as in the firſt, that 1 added to 2 (the independent motion it acquired by the firſt time's fall,) makes 3, and this added to the 1 it fell in the firſt time, makes 4 (ſquare of 2 as before).—When it arrives here, it has by the ſame reaſoning acquired an independent motion of 8, in 2 times, which is 4 in 1, and that added to 1, which it falls by the action of gravity in each time makes

5, which

5, which added to 4 it has fallen in the preceding times, makes 9, the ſquare of 3.— Therefore at every period the independent motion acquired by the falling body is in geometrical proportion to the times, and double the ſquare root of the diſtance fallen through, agreeably to the table below.

If the firſt time be fixed to a quarter of a ſecond, the body will fall in the following progreſſion.

TABLE.

Times.	Fall in each Time.		Fall at the different Periods.		Independent Fall acquired.	
	F.	I.	F.	I.	F.	I.
1ſt — $\frac{1}{4}$ of ſec.	—— 1	$\frac{1}{8}$	—— 1	$\frac{1}{8}$	—— 2	$\frac{2}{8}$
2d — $\frac{1}{4}$	—— 3	$\frac{3}{8}$	—— 4	$\frac{4}{8}$	—— 4	$\frac{4}{8}$
3d — $\frac{1}{4}$	—— 5	$\frac{5}{8}$	—— 9	$\frac{9}{8}$	—— 6	$\frac{6}{8}$
4th — $\frac{1}{4}$	—— 7	$\frac{7}{8}$	—— 16	$\frac{16}{8}$	—— 8	$\frac{8}{8}$

N. B. The laſt column ſhews the actual and independent velocity the body has acquired at the end of each term; which is what it would continue to move with per $\frac{1}{4}$ ſecond, if the action of gravity were removed. So that a body having fallen through 16 ft. 2 in. in one ſecond, it would acquire a velocity of 32 ft. 4 in. per ſecond, independent.

Thus it is demonſtrable, that if a number of balls of any weighty matter were placed on the wall of a tower, or other eminence, and were to be puſhed forward in ſuch a manner that they ſhould ſucceſſively fall, as faſt as one gave place to the other, yet would they run from each other, as deſcribed by plate 1. fig. 1. For if the ball (a) were to fall to b, and then another took it's place;—while this laſt was falling through the firſt ſpace, the firſt ball would reach to c, and ſo ſucceſſively to d, f, &c.

SECTION II.

Of the Deſcent of Water, and the effect of it's paſſing through Tubes, and other Veſſels.

WATER, like other heavy bodies, would obſerve the ſame rules in open ſpace; and therefore if it be poured from an eminence, though the ſtream may be continued from the veſſel, yet would it not remain ſo during it's fall, but the firſt eſcaping, would fall away from the laſt, and increaſe in diſtance from it till it reached the end of the fall. The reaſon why it ſhould be otherwiſe, when confined, and falling through a tube ſuch as plate 1. fig. 2d. will ariſe from the following conſideration; that, although by filling the reſervoir on the top, and the water falling into the tube, it's parts would have a tendency to divide, and have ſeparate motions, according to their ſucceſſive times of begining their falls; yet if the tube be regular, and equal in it's bore, no portion of the column of fluid can go faſter than another; becauſe the preſſure of the ſurrounding atmoſphere would keep it together, and not ſuffer by it's diviſion a vacuum between it's parts:--therefore in ſuch a caſe, inſtead of falling out of the lower end of thetube, with a velocity ſufficient by an equable motion (if gravity were ſuſpended) to carry it, through a ſpace *double* the height of the tube, in the time a body would fall the length of it, as ſhewn above; it's impetus would only carry it through a ſpace *as long* as the tube, in the ſame time that it would fall the length of it; for it would all fall with the *medium* of the velocity of a ſolid body in falling that length. But this defect of natural motion may be remedied, by forming the tube like a trumpet, and of the proportion of fig. 3d. plate 1. or of any other exceeding it in it's upper parts;—for then it will contain ſuch quantities of a fluid in it's different proportionate heights, as are equal to the ſpaces through which they would fall by their natural weight, (making up in bulk what is wanting in length); if therefore, there were a ſupply at top, by a proportionate deſcent of the fluid in each part the motion would be agreeable to the deſcent of a body falling by accelerated motion; for as much would deſcend by it's fall from a to b, as from b to c, or c to d, &c. which are the natural falls, and the fluid would have no tendency to divide in the tube. In this caſe, therefore, the iſſuing of a fluid through a hole in the bottom of the veſſel would be with a velocity equal to that which a heavy body would acquire by falling through a ſpace equal to the height of the fluid, and which, as ſaid before, is double the *medium* of velocity it had in falling, or in other words, ſufficient independently to carry it through double the ſpace it had fallen in the ſame time.—This reaſoning is new, (at leaſt to myſelf, not having met with the cauſe of the effect before mentioned any where explained:) but the common doctrine of bodies falling in open ſpace is accepted by moſt philoſophers, and was that of the excellent Sir Iſaac Newton, who in the proceſs of his inveſtigation, lays it down as a law of Nature, " That the comparative " impetus or force of bodies in motion, is, as " the velocity with which they move multi- " plied into their quantities of matter."

I ſhould have thought it highly aſſuming in me, to contradict in any terms ſuch great authority; but having occaſionally had ſome caſes to inveſtigate, in which this law was concerned, and not finding it to ſolve in a very ſatisfactory way the doubts I had to determine, and conſidering a blind adherence to any doctrine, without a conviction of the juſtneſs of it's principles, as a bar to knowledge and diſcovery, I thought it right to expreſs my objections to that doctrine, hoping, that it may be the means of exciting to a further inveſtigation ſome perſon better able to purſue it, and tend to ſettle a ſubject, ſo worthy, by it's general concern, the attention of all.

In this having had occaſion to refer to ſome books which I had never before conſulted on the ſubject, I ſoon diſcovered that the opinion which I had taken up (and which was, that the comparative force of bodies in motion was as the ſquare of their velocity, multiplied into the quantities of their matter,) was not new, as I had conceived, but had been hinted by Huygens, and adopted by Leibnitz and his followers; who in many publications had ſupported their opinions againſt the Newtonians, with arguments and experiments many of which I had previouſly conceived.—

Some of theſe I conſider as concluſive but, as they are in print, I ſhall not here repeat them; offering only ſuch as appear to be new, and are convincing to me of the juſtneſs of the latter doctrine, at leaſt till other arguments are preſented than thoſe which I am at preſent acquainted with.—Preparatory to this, the following conſiderations and poſitions are neceſſary, and require the aſſent of the reader.

SECT. III.

Of the various kinds of Forces, and their Effects on Mechanical Powers.

POWERS, or forces, in a mechanical ſenſe, are of three kinds, under two diſtinct heads; firſt, the inactive, reſiſting power, or *vis mortua,* which admits of no variation, neither can have any thing in it relative to quantities of matter, ſpace, time, or velocity, (which laſt is a compound of the two preceding it) for ſhould it yield to the force applied, it's diſtinction alters.---Such is the reſiſtence which a table, wall, &c. makes to any power, or thing placed on, or againſt them.

Second, That kind of active force, or *vis viva*, which ariſeth from the percuſſion or ſtroke of any body in motion, (however that motion might have been received) whoſe quantity of power or force muſt be a certain compound of it's quantity of matter, and quantity of velocity at the time of percuſſion, without regard to the ſpace it might have before paſt over, for that can have no reference to a *poſt* action; thus, any independent body, having a certain degree of motion once given it, would continue to move equally till it were oppoſed by ſome other body, againſt which it's action would be equal, in whatever point of it's courſe they ſhould meet.

Third, That other kind of *vis viva* which is produced by gravity or otherwiſe, on two bodies connected by ſome machinery to each other, and is compounded equally of quantity and ſpace, without regard to time or velocity.—Such is the *action* of matter yielding to gravity, or the attraction of the earth, and conſtantly expending itſelf on ſome oppoſing power; in which caſe it is evident, that a certain power lies in a given quantity of matter, in reſpect to a given perpendicular diſtance, however faſt or ſlow that power be uſed, and is ſubject to a number of variations by the uſe of mechanical powers; thus, if I have an hundred pounds weight of any matter to lift through a foot of ſpace in a line directly contrary to the earth's attraction, (or upwards,) I may effect it by the leaſt poſſible quantity more than an hundred pounds weight, moving by the action of gravity through the ſpace of one foot downwards, either by means of an equal balance, by a ſimple pulley, or by a double plane of any equal inclination.

Ex: In the double inclined place E C F plate 2d. fig. 1ſt. ſuppoſe the weight A to be one hundred pounds, from which let a line be carried over the pulley C and faſtened to B, (a weight of little more than one hundred pounds) it would, (if the planes E C and C F were equal,) raiſe the ball A up the inclined plane, juſt as much as B deſcended, and no more, which ſuppoſe to be a foot perpendicular deſcent, A would be juſt ſo much raiſed, and the ſame would happen by inverting the planes, and making uſe of two pulleys inſtead of one, as in figure 2d.

Or it may be effected by the action of little more than fifty pounds through two feet deſcent, or twenty five pounds through four feet deſcent by means of any of the mechanical powers,---converting the equal balance into a lever, the ſingle pulley into a compound ſet, or the equally inclined planes into unequal ones; thoſe powers, therefore which are made up of *matter* and *ſpace*, ſuch as one hundred pounds raiſed one foot, fifty pounds two feet, or twenty-five four feet, (whether this be an hour, a day, or month in the operation, or whether the motion or velocity be ſwift or ſlow) are a balance to each other, and may be conſidered as equal, and expreſſing the ſame power.

SECTION IV.

Of the Application of the preceding Conſiderations to prove the Leibnitzian Doctrine of the Percuſſion of Bodies in Motion.

THESE premiſes being allowed, (and I think they are incontrovertible) it will be no difficult matter to prove, that the comparative force, momentum, impetus, or powers of equal bodies in motion, are as to the effect of their *percuſſion*, as the *ſquare* of their velocities, and conſequently that the ſame powers of bodies of unequal weight, are as the *ſquare* of their velocities multiplied into their reſpective weights, or quantities of matter.—For it is a generally well known and acknowledged fact, that a pendulum, (ſuch as is uſed in clocks,) raiſed to any given height, and let ſwing freely, (ſome ſmall allowance being made for the reſiſtence of the medium in which it vibrates and the friction of it's axis,) will riſe by the power gained in it's fall, to the ſame height on the other ſide of the perpendicular, then return back to it's firſt place, and continue to make equal vibrations ad infinitum.—So would it happen to a ponderous ball, (the ſame allowances being made for friction, &c.) if it were let run down ſuch a double inclined plane, as fig. 2. plate 2. for inſtance ſuppoſe from B; the power it would acquire by the deſcent to E. F. would be ſufficient to raiſe it an equal height on the oppoſite plane, to which it's direction would be diverted, provided it be contrived by curvating the angle at E F, to make that diverſion gradual and free from ſhock; and this would happen in the ſame manner let the ball run from whatever part of the inclined plane it may; correſponding exactly with the action on the inclined plane, conſidered as a balance when the figure was before treated of. It cannot be objected to this reaſoning, that the ball's aſcending on one plane, by it's deſcent on the other from an equal height, affords not a fair argument, as the power is not communicated to a ſecond body; for whatever power exiſts in any body may be communicated in equal degree, provided that by the *contrivance* or mode of communication none of that power be loſt and although no ſuch contrivance or mode can be found out, yet ſuch defects never make part of calculations of this ſort. Nor can it be fairly objected, that the raiſing a body in motion is not equal to the raiſing an equal body deprived of it to the ſame height; for in this caſe we are conſidering the effect of that motion, which is the raiſing the ball, and without which it would not be raiſed;—the height therefore to which it is raiſed, expreſſes the power, as in other mechanical operations before ſtated.

We may here juſt mention that bodies perfectly elaſtic, falling on ſolid bodies, and water ſpouting upwards from a hole in the ſide of a veſſel, will both riſe to the height of their ſources of action.

If we now take a retroſpect of fig. 1. plate 1. and the explanation of it, we ſhall there ſee, that the comparative, actual, and independent velocity of a falling body, is not as the height, or diſtance through which the body falls, but as the *ſquare root* of that diſtance, as marked in the laſt column;—therefore preſuming it is ſhewn above, that the powers of equal bodies are as the heights from which they fall, thoſe powers muſt be as the ſquares of thoſe velocities, which are as the heights. —And the further to confirm the preceding doctrine, I will endeavour to ſhew in it's true light the chief argument which has been advanced againſt it, and, which confounding the two diſtinct powers beforementioned, is founded on the following principle;—that if to a lever two unequal weights be applied, at ſuch diſtances from the fulcrum or center of ſuſpenſion as that they ſhall balance each other, ſuch as 4 pounds at 1 foot diſtance, balanced by 1 pound at 4 feet from the fulcrum, they will not only be in equilibrio to each other when they are at reſt, but alſo when put in vertical motion, which it has been ſaid could not happen if the ſquare of the velocity meaſured the force. For in the lever fig. 3. pl. 2, if the weight A of 1 pound be put in motion downwards to C, it will raiſe the weight B of 4 pounds to D—Now as the ſquare of 1 (the velocity of B) is 1, which multiplied by 4 (the quantity of matter) is 4;—and as the velocity of A is 4, whoſe ſquare is 16,—it ſhould therefore ſeem, that according to the new doctrine theſe numbers expreſs the comparative powers of each when in motion, and that therefore that of A, would overcome that of B, and inſtead of vibrating as it is found to do, would increaſe in velocity till it reached the loweſt point which the length of the lever would permit: but this reaſoning gives way when we conſider that no motion can take place in A, without it's acting and expending it's power at *the ſame inſtant* on B, therefore no free motion through any quantity of ſpace, or *percuſſion*, can poſſibly happen; in which caſe no *velocity* can be eſtimated in the calculation of the power or effort, for the ſame reaſon—If I have a ball of matter of any given weight, (4 pounds for inſtance,) whoſe power in it's deſcent through 49 feet of ſpace, I can make uſe of, and I ſuffer it to fall through the whole, it's power by percuſſion at the end of it's fall may be eſtimated as the ſquare of 7, (it's velocity,) multiplied into 4, (it's weight or quantity of matter,) which is 196.—Now, if inſtead of making uſe of the percuſſion acquired by it's going through the whole fall, I take it at twice;—as firſt, it's percuſſion or force when fallen through 25 feet it will then have acquired a comparative velocity equal to 5, whoſe ſquare 25, multiplied by 4 the weight, is 100; and by the ſame reckoning, it will by running through the remaining 24 feet acquire a momentum equal to 96, which together is as the firſt force gained by the whole fall. And ſuch will be the reſult, if we calculate the percuſſion or force of the fall through each foot of the ſpace, ſeparately;---for the ſquare of 1, is 1, and multiplied by 4, is 4, and this on each of the 49 feet makes together 196, as before. What is moſt obſervable in this, is, that in calculating the percuſſion or force of the body in the fall of 49 ft. taken together, the ſquare of the velocity is 49; but as the diſtances of the ſeparate falls decreaſe, notwithſtanding the effect produced on the whole of the falls be equal, yet the velocity has leſs and leſs proportion in the calculation, till it comes to that point in which there is no difference between the action of the power and effect, (as to time and ſpace;) as is the caſe with the lever or ballance, and being a power of the third ſort here enumerated, in which velocity and percuſſion are quite out of the queſtion; therefore, although this doctrine be not confirmed by the action of weights ſo ſuſpended, yet that is by no means an argument againſt it.

It may not perhaps be deemed ſuperfluous juſt to remark, that, as the comparative powers of bodies in motion are as the ſquare of their velocities, ſo are the comparative velocities of a body, (as obtained from any powers.) as the ſquare root of the powers. This indeed ſtands or falls with the preceding doctrine, for it's beſt proof depends on the ſame argument.--- And what teſt have we of the degree of power or force in any body, but the quantity of work it will do, before it's power be expended? therefore whatever power will raiſe a certain weight of matter 4 feet muſt certainly be 4 times as great as that, which can raiſe the ſame but 1 foot. Now it has been ſhewn, that a body falling through 1 foot of ſpace, will raiſe an equal weight 1 foot and a body falling through 4 feet will raiſe an equal weight 4 feet and ſo on; conſequently, their powers or forces are as the diſtances fallen through; but as is above ſaid, their velocities will be but as the ſquare *root* of theſe powers, the firſt being 1 and the latter 2, and the ſame rule obtains in every diſtance*.

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* I am aware that much has been written, to ſhew, that the *time* required to expend the power contained in any independent body in motion, by it's percuſſion or impulſe on another, varies according to the degree of it's velocity, and therefore that this difference of *time* ſhould be taken into the account of it's action or power; but waving the very intricate diſcuſſion and experiments which have been offered on the ſubject, the queſtion may be reduced to the following ſimple one;—Will any two independent non-elaſtic bodies meeting in contrary directions, and having quantities of matter and velocities, which by the above rules conſtitute equal percuſſion or impetus, inſtantly deprive each other of motion or not? I have no opportunity of making the experiment as it ought to be tried, but have little doubt of it's ſucceſs in proving the above doctrine.

If this be clear to the conviction of the reader, he will have cauſe to admire the unity and ſimplicity of the laws of nature and their operations; as, if it were otherwiſe than I have ſtated, there would be in effect, a diverſity unaccountable, and without cauſe; for what reaſon can be aſſigned, why a body, falling through one meaſure of ſpace, (without acting on any thing in it's fall, ſo as to expend it's power,) ſhould not double that power by falling through two ſuch meaſures, juſt, as a body of double the weight of another, has double the power by falling through the ſame height.

SECTION V.

Of Water on overſhot Wheels.

BEFORE we apply this reaſoning to the power of a fluid ſpouting through a hole in the ſide of a veſſel, or rather to it's re-action on that veſſel, let us conſider the effect of waters on overſhot wheels; and in theſe, we ſhall find the power in exact proportion to their ſize, or the height of the water's action.---For inſtance, if I have a ſtream of water, which will afford a conſtant loſs of 1 gallon per ſecond, and have alſo a ſufficient fall to allow it to paſs over a wheel of 14 feet diameter, which is equal to 44 feet in circumference, and if there be a bucket capable of containing a gallon, in each ſoot, and the wheel revolves in 41 ſeconds; couſequently if the buckets are contrived to diſcharge themſelves as they arrive at the lower part, and are gradually ſilled at the top of the wheel, the expence of the water will be according to the above allowance of 1 gal. per ſecond, and there will be 22 gals. conſtantly acting by their weight on one ſide of the wheel, and none on the other; but. as the perpendicular bearing of the weight of each bucket will be at different diſtances from the perpendicular of the wheel's center, it will be neceſſary to delineate ſuch a wheel, in order to make it's action clear to the underſtanding as we proceed.

Let A B C D, fig. 1. plate 3. repreſent ſuch a wheel having 41 buckets, each containing 1 gal. of water, and ſo contrived, that there ſhall be no vacant ſpace between them. and that they ſhall not diſcharge themſelves, till they arrive at the bottom of the wheel, or near it; then it will be found, that the aggregate action of the buckets will be as centered on the point E, and will therefore be a balance to two thirds the weight of the 22 gallons if ſuſpended at B, (which is 1 and an half the diſtance of E from the center of the wheel;) this weight therefore, which I ſuppoſe to be 132 pounds, it would nearly raiſe, at the rate of one ft. per ſecond.---Now ſuppoſe the wheel to be double the ſize, as fig. 2. it will then contain 88 buckets on it's circumference, or 44 gals. acting by the weight on the point E, and would ſupport 2-thirds of the weight of 44 gals. ſuſpended at B, which is equal to 264, double 132 pounds.) If each wheel therefore revolves at the rate of 1 ft. circumference per ſecond, (which it muſt do to keep the expence of water equal) the power of ſuch wheels muſt evidently be in exact proportion to their ſize,---or in other words, the power of deſcending water by it's application to ſuch wheels, is in exact proportion to it's deſcent, the expence remaining equal,

SECTION VI.

Of the Re-action of Spouting Fluids.

WE come now to treat of the effort or power produced by the preſſure of a continued body of fluid, contained in one veſſel; in which is concerned, that certain and evident law in hydroſtatics; " that on equal ſurfaces the quantity of ſuch preſſures is in geometrical proportion to the height of the ſuperincumbent fluid, without regard to the ſize of the containing veſſel," " and alſo," that (on account of the yielding property of ſuch as are pretty perfect, occaſioning a re-action from one ſide or part of the veſſel to it's oppoſite,) the preſſures are equal in every direction.——Thus, if a veſſel 10 inches deep be full of water, the preſſure on every ſquare inch of the bottom, will be equal to the weight of 10 cubic inches of water, and on every ſquare inch of the ſides will be equal to the weight of as many cubic inches of water as are perpendicularly above it; the direction of which preſſure will be at right angles to the part on which it acts:---So in a veſſel 10 feet ſquare, and 20 deep, full of water, there will be the weight of 20 cubic feet of water on every ſquare foot of it's bottom; and on each ſquare foot of it's ſide, whoſe center ſhall be 16 feet from the top of the fluid, there will be the weight of 16 cubic feet of water, preſſing in a lateral direction againſt it.---If, therefore, a hole of a foot ſquare be made at this depth, and ſome independent body be put againſt it to hinder the water from running out, there will be the preſſure of 16 cubic feet of water preſſing more on the oppoſite ſide than on this; and if the veſſel be put afloat, or in any ſituation free to move, it will require the action of ſuch weight to keep it from motion; if this preſented body be removed, the water will iſſue with a velocity equal to that which a body would acquire by falling from the height of 18 feet which we will call 8 feet per ſecond; now as the veſſel is ſo large in proportion to the quantity of water running off, and we ſuppoſe it by ſome ſupply kept conſtantly full, the preſſure on the foot oppoſite will not be much diſturbed or leſſened; if therefore the veſſel were to move in the direction of the ſuperabundant preſſure, and with the whole velocity of the iſſuing water, all the power of ſo much iſſuing water will be expended on the veſſel, and it will raiſe a weight of 16 cubic feet of water through 8 feet of ſpace, which is equal to the expence and height of the fluid.---If this hole be ſhut, and the veſſel remaining in the like circumſtances, a hole of the ſame ſize be opened in the ſide, whoſe center ſhall be 1 foot below the ſurface of the fluid, it will in the ſame proportion run 2 feet per ſecond, and there will be a ſuperabundant preſſure on the oppoſite ſide, equal to the weight of 1 cubic foot of water; if then, as before, the motion of the liquid be ſuppoſed to expend itſelf on the veſſel, it will move it 8 feet in the time that ſo much is run out as above ſtated; and will raiſe, by the contrivance of a pulley, the weight of 1 ſquare foot of water through 8 feet of ſpace: which is one 16th of the effect of that iſſuing from the lower hole, and is in exact proportion to the height.

It is true the motion of the veſſel would not be as here ſuppoſed, but that part of the power or motion of the iſſuing water, which was not expended on the veſſel, would be retained by it, and might be uſed on any other body oppoſed to it's motion. And if the veſſel were kept ſtationary, the power of the iſſuing water would remain entire in itſelf, and would (leaving the waſte of power by the daſhing of the water out of the calculation) be ſuch, as any other body in motion would have, agreeably to the law we have ſupported, viz. As the ſquare of it's velocity multiplied into the quantity of it's matter, which is in geometrical proportion to it's height.—For, if we eſtimate it's velocity from the upper hole as 1, and quantity 1,---the power would be 1.---If from the lower hole as 4, the ſquare of 4 is 16, for every quantity equal to the above, which is 1.---It is alſo evident, that 4 times more would iſſue from the lower hole than from the upper hole *in the ſame time;* but then it's work will be 4 times more than *it's proportion* in the ſame time, and we are not to meaſure work by time, but by the quantity done, or expence of power applied to it:---For the ſame reaſon, if I have 3 hundred weight of matter to lift through 1 ft. of ſpace, and have three men to effect it, each of whoſe ſtrength is equal to lift 1 hundred weight through that ſpace, I may accompliſh it either by their joint effort on a ſingle pulley, or their alternate efforts by means of a three-fold pulley; and although the latter means will be three times as long in action as the firſt, yet the power expended, and the effect produced muſt be allowed to be the ſame in both caſes;---and arguments before urged tend to prove the ſame thing.

A ſimilar reaſoning holds, if inſtead of the large veſſel mentioned above, a regular ſmall tube with a reſervoir at the top were uſed; for although in this caſe the water would not iſſue from a hole near the bottom, but with half the velocity actual to the preſſure of ſuch a height, (for reaſons before given;) yet the relative and comparative action of the water iſſuing, would be in proportion to the ſquare of it's natural velocity and expence, or to it's height and expence, which keeps the ſame proportion; and this is demonſtrable, whether we found the conſideration of it on that certain law, that action and re-action are equal, and in contrary directions, or trace the mere action of the column of fluid preſſing on the part immediately oppoſite to the aperture of the ſame ſize, and the motion of the iſſuing water, for they are dependent on each other.

Thus, if ſuch a tube as A, B, fig. 4. pl. 2, one inch ſquare in cavity, and 50 inches long, contained water to the height of 49 inches from B, the preſſure on a ſquare inch about that point would be equal to the weight of 49 cubic inches of water, and the preſſure on the ſquare inch oppoſite to it would be (from their mutual action) the ſame; therefore the tube would have no more tendency to move one way than another; but if a hole of 1 inch ſquare be opened at B, the preſſure would be taken off from that ſide;---and if the veſſel were large enough, to ſuffer the water to run off with it's natural velocity from the aperture, it would alſo continue to preſs with it's natural weight on the ſquare inch oppoſite, and act as in the former mentioned veſſel; but as that is not the caſe, whatever preſſure be deſtroyed, and becomes wanting to force the water out at B, muſt alſo be wanting to preſs towards C;—if then we ſuppoſe each of theſe to be but half it's natural weight, it will be juſt as if the column were divided, and a weight, equal to that of 49 cubic inches of water, were requiſite to keep the balance againſt the re-action of that iſſuing at B, which inſtead of running off with the velocity of $\frac{1}{7}$ (the ſquare root of 49,) would do ſo but with the ſquare root of 24 and an half which is a trifle leſs than $4\frac{95}{100}$, the power in the action and re-action of which quantity is but as 24 and an half, whereas if the veſſel were large it would be as 49. The power or quantity of work this will do will therefore be as the expence and fall of water in the overſhot wheel, and all other caſes.

SECTION VII.

Of Barker's Mill.

THIS, therefore, determines the power of Barker's mill, a contrivance intended to direct the re-action of ſpouting water to the purpoſe of grinding corn or other work, and which is recommended for it's ſimplicity, and ſmall degree of friction, (having no wheels,) but turning on an upright ſpindle, paſſing through an erect and croſs tube; as in the fig. [pl. 4. fig. 1.] the tubes A, B, and C D receiving water from a reſervoir; at the extremity of each arm, and on oppoſite ſides, holes are opened, and by the re-action of the water ſpouting through them, the whole continues to revolve, turning a millſtone fixed to the ſpindle at E.

If this machine be prevented from turning, the power of the water iſſuing from the holes will be, as before ſhewn, a compound equal to the height and expence of the water, juſt as in overſhot wheels * ; but for

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* The philoſophical reader will be here ſtruck with the appearance of an acceſſion of power to this machine, by the action of the centrifugal force, co-operating with the weight or preſſure of the water in the upright tube, by which the work effected ſeems to increaſe beyond the quantity of the power applied, but the fallacy of this appearance will be ſeen, on recollecting, that this machine is no other than an inverted centrifugal pump, and as in that machine, the reſiſtence to the power applied to work it, proceeds from new matter conſtantly ariſing to be put in circular motion, which is in exact proportion to the water raiſed and the height it is raiſed to—ſo in this the diminution of the power of the machine by freſh matter falling into the croſs tubes to be put in circular motion is in exact proportion to the ſeeming increaſe of power by the centrifugal force acquired: therefore although the work done by the machine be increaſed by the centrifugal force acting on the water, yet the expence of power keeps exact pace with it.

for their action to be equally great on this machine, it is neceſſary, that the whole motion of the iſſuing water be expended on it, which cannot well be done, but by receiving it's action on ſome moveable body, which will communicate it to the former, and that may be effected by applying a recipient horizontal wheel, and connecting it to the croſs tube by a vertical wheel.—This it muſt be allowed, complicates the whole to ſuch a degree, that it leaves it no other preference to the overſhot method, but that a wheel (or circle formed by the revolution of the croſs arms,) of a moderate or even ſmall ſize will do the ſame work, as a larger one at an equal expence of water, as to quantity and fall. For in the former, if the water acts by it's weight only, it is certain, that the wheel muſt be equal in diameter to the ſall of the water; but in the latter caſe, the action does not depend on the length of the croſs arms, but on the height of the upright tube, or rather that of the water contained in it, and even this part may be improved by connecting the pipe, which brings the water for it's ſupply to the upright tube, by a water-tight joint, guarded by a leather flam, as in the pl. 4. fig. 2. which alſo repreſents the before mentioned addition, intended to communicate the whole action of the iſſuing water to the performing the work of the machine;—but that this may compleat the whole intention, it will be neceſſary, for boards to be ſo fitted within the trough of the recipient wheel, that, by the action of the water, they may riſe and obſtruct the current, which might otherwiſe take place round it, and thereby waſte part of the power. No ſuch contrivance as this laſt is neceſſary, to prevent the loſs of power, by the yielding of overſhot wheels to the impulſe of the deſcending fluid, for they are in themſelves admirably adapted to avoid ſuch loſs, which will appear if we conſider the different motions as to perpendicular fall, of the water contained in each bucket, whilſt the wheel revolves.

Deſcription of fig. 2, plate 4.

A B C D	—The frame of the Machine.
F G H I	The upright Tube, The croſs Tube, } Correſponding to thoſe of fig. 1.
K L	The recipient wheel, which, by the action of the water iſſuing from the croſs tube, is turned (in a direction contrary to it) round the Spindle S, on which it is ſupported by a Nut under the Block R, being kept in its horizontal poſition by working round the Spindle by a Collar in the centre of the Wheel M.
M N O	Wheels, which connect the croſs Tube and recipient Wheel, ſo as to reſolve their action to the ſame effect, the axis of the Wheel N being fixed to the Spindle S.
P	The upright Spindle, which paſſing down through the upright tube and turning on the head of that round which the recipient wheel turns, revolves with the Machine and works the Stones.
Q	A cloſe Pipe, which brings the water to the Machine, and ending in the Box L, is there connected with the upright Tube by an iron Collar, guarded by a leather Flam, which hinders the water from eſcaping, but through the Tube.

SECTION VIII.

Of the due proportion of Reſiſtance, applicable to a Machine worked by the third kind of Power, or Percuſſion, in order to cauſe it to effect the moſt Work.

BUT leaving at preſent the conſideration of this particular, it will be proper to inveſtigate, and form our judgment on a doctrine generally received, which is, that in order for any power to produce the greateſt effect in a given time, by it's application to any given machinery, it is neceſſary that it ſhould move the part on which it acts, with one certain degree of it's own independent motion.

Thus far muſt be true, for if either the machinery remained unmoved, or had a velocity equal to the power, that power could effect no work by it; the due proportionate difference muſt therefore lie between thoſe extremes; and that difference, it is alledged, ſhould be juſt double in the power of what would be a balance to the weight, or work done; or in other words, a machine ſhould never be loaden with more or leſs weight or oppoſition to the power, than juſt half of that, which would be an equal balance to it.—And the reaſon which has led to this poſition, is: that under theſe circumſtances any power (unlimited in it's duration,) will effect that portion of work in the ſhorteſt time; and this ſo far as it is confined to ſuch particular circumſtances, agrees with the theory laid down in the preceding ages, as will be clearly explained in the diagram, Plate 5, in which A B is intended to repreſent a lever, (or rather wheel and axle in peritrechio,) having wheels imagined at each diviſion of the lever, whereby to ſuſpend the weight C, to repreſent the power acting on A, the raiſing which repreſents the work effected, and as their weights are equal, the firſt being hung on at C, will be juſt a balance to the latter.

As it has been ſhewn, that when a body falls from any height, it's ſtrokes or percuſſion are in ſimple proportion to the heights, and it's velocities as the ſquare root of the heights, ſo, if ſuch a body be attached to any machinery, &c. whereby part of it's power in the fall may be expended, the difference between the power exerted, and that expended, will remain in the body,, and it's average velocity throughout will be conſequently as the ſquare root of that difference.—Suppoſe therefore the weight at A were to be raiſed, no motion could take place: but if it be moved one diviſion further from the center, (and no friction ſuppoſed,) it would then deſcend to F 101, in which time it would raiſe the weight A 100; the difference between them, therefore, repreſents the comparative difference between the power expended, and the work effected, which is 1, whoſe ſquare root 1, is the velocity with which it would move, compared to that it would acquire by being placed further onwards; which ſuppoſe to be at 125;—that number is the diſtance to which it muſt fall, to raiſe the weight to 100.—The ſquare root of the difference being 5, is therefore the velocity of the weight of falling from this part, and ſo on as expreſſed by the figures in the diagram.

This being the caſe, if the diſtances through which the power muſt deſcend, (to bring the lever into the direction D E as before,) be divided by the comparative velocities from each place of ſuſpenſion, the products will be the comparative times in which the ſame weight will raiſe the weight A as required, and this time will invariably be found to be leaſt, when that power is double the diſtance from the center or fulcrum of the lever of that point at which it would balance the weight.

Deſcription of Plate 5.

Plate 5 includes three diagrams expreſſed by the three different kinds of lines, viz. full, ſhort dots, and long dots, the firſt being included in the ſecond, and both in the third. In each, the ſpaces above their reſpective croſs-lines ſhew the diſtance through which the power (expreſſed by the Weight C) muſt fall, before as much be expended as is equal to raiſing the Weight A by the whole fall of the lever. The length of the ſeveral lines below the croſs-lines, ſhews therefore the exceſs by which the average velocities are found (by extracting the Square Root). The lengths of the ſeveral lines are ſelected to anſwer to ſuch velocities as may be expreſſed by whole Numbers.

The times of falling through the diſtances as expreſſed by the *full lines* in the diagram, Plate 5, will be according to the following proportion :

Diſtances to fall to bring the lever to the angle of 45 deg.	Comparative velocity.	Comparative times.
101	1	101
125	5	25
149	7	$21\frac{2}{7}$
164	8	$20\frac{1}{2}$
181	9	$20\frac{1}{9}$
200	10	20 leaſt
221	11	$20\frac{1}{4}$
244	12	$20\frac{1}{3}$
269	13	$20\frac{9}{13}$
296	14	$21\frac{1}{7}$

Nor would this rule vary, with the alterations of proportion between the power and weight, or the angle to which the lever deſcends ; for if a weight of one quarter of

A were hung on at 400 it would balance, then would it be found that at 800, (double according to the rule,) it would fall to the angle of 45 degrees in the ſhorteſt time, as follows:

500	10	50
596	14	$52\frac{4}{7}$
689	17	$50\frac{9}{17}$
724	18	40^{9}
761	19	$40\frac{1}{19}$
800	20	40 leaſt
841	21	$40\frac{1}{21}$
884	22	$40\frac{4}{22}$
929	23	$40\frac{9}{23}$
1025	25	41
1076	26	$41\frac{5}{13}$
1184	28	$42\frac{8}{28}$
1300	30	$43\frac{1}{3}$

or if it had to fall to the lower angle G H, or any other, it would be found to make no difference in the proper point of ſuſpenſion, for the comparative velocities and diſtances would be as marked in the *dotted lines*

lines in the diagram, which will ſhew that the weight will fall to 1584, in the ſhorteſt time on the ſame line as before, according to the following ſcheme.

900	-	10	-	90
996	-	14	-	$71\frac{5}{7}$
1200	-	20	-	60
1425	-	25	-	57
1584	-	28	-	$56\frac{8}{14}$ leaſt
1700	-	30	-	$56\frac{2}{3}$
1889	-	33	-	$57\frac{8}{11}$

Hence it appears clear, (without the uſe of Algebra,) that under all circumſtances, in which the duration or continuance of the power is not conſidered, ſuch as that of a machine worked by a wheel limited in ſize, receiving water without percuſſion, and paſſing through an indefinite fall or power acting by gravity, to draw a weight up an inclined plane, or work pulleys, &c. the greateſt advantage is made of the power when the machinery on which it acts is laden (by work and friction,) with no more than half of that which would be a balance to it. But to extend the ſame law to caſes in which the continuance of the power is limited, and of which every advantage might be taken, ſuch as any certain quantity of water in a reſervoir, acting by it's deſcent through a limited fall, is uſing that power to great waſte, and ſuffers ſuch water to run off with a conſiderable part of the power it acquired in the fall. For if the fall of the weight from 401, were limited to the line I K, and it were to be hung on the wheel, ſuppoſed to be on the lever, at 800, it would fall with a velocity twice as great as if it were hung at 500; but if it were diſcharged at the line I K, another power equal to it muſt at the ſame inſtant take it's place at 800, ſo as to bring the lever to the line E. In the ſame time that this would be effected an equal weight at 500 would have reached the *dotted line*; therefore, although at 800, twice the quantity of power would have been expended, yet at 500, it would have wanted but one-fifth of effecting the ſame work in equal time, and, by various applications of machinery this work might be varied *in infinitum* by an exchange of quantity of work, for velocity or quickneſs of time, and *vice verſa*.

If we apply this doctrine to overſhot wheels, receiving water from a reſervoir circumſtanced as above, we ſhall find that the ſlower the wheel goes, ſo as it be faſt enough to keep on it's motion, the more work will be done at the ſame expence of power. Experience it is true, oppoſes itſelf in ſome meaſure to this doctrine, but I think the apparent contradiction between theory and practice may be ſatisfactorily accounted for, not only by the friction of the machine making a conſiderable difference in favour of the uſual method but alſo on the following principle.

SECTION IX.

Of Fly Wheels, and their Effect on Machines.

IN all machines to which water is applied, the work intended to be performed by it is not perfectly regular and uniform, and therefore a Fly Wheel, or ſome ſubſtitute for it, is neceſſary to regulate the motion and prevent a ſtoppage, when any extraordinary obſtacle to the movement occurs in the work.

When the motion is very ſlow in a water wheel it cannot act as ſuch a ſubſtitute, but if it has conſiderable velocity it may. The power loſt by ſuch velocity is, therefore, not ſo great a diſadvantage as the loſs of the wheel's action as a regulator.

Whether then, by the application of ſuch a fly that office might be taken from the wheel, and an advantage gained by it's being made to move ſlower than is generally done, is certainly worth enquiry; and to proceed in it we muſt inveſtigate the laws on which the action of ſuch Fly Wheels depends, in which it is more neceſſary to be particular, as their uſe in moſt kinds of machinery deſerves ſuch attention.

As action and re-action are equal, and in oppoſite directions, and it being impoſſible that an effect can be greater than it's cauſe, therefore a Fly Wheel can only regulate, not increaſe the power of any machine; it's uſe being to act as a reſervoir to receive the redundancy of the power applied, when either by a variation in it, or the work, the proportion of the former to the latter ſhall be greateſt; and to return that power when it bears the leaſt proportion to the work.

It has been before ſhewn, that the power or force of bodies in motion by their percuſſion, is as the ſquare of their velocities multiplied into their quantities of matter; conſequently, that the velocities of bodies as acquired by power impreſſed, are as the ſquare root of the powers divided by the quantities of matter;—if then, that power be derived from a body in accelerating motion (ſuch as a Fly Wheel receiving motion from a falling body,) the Fly will alſo receive an accelerating motion in ſuch proportion, but it's degree of motion at the ſame diſtance of falling will be leſs.

Thus, ſuppoſe one pound weight of matter having fallen through 16 feet, the velocity it has acquired at that point we will call 4; the ſquare of which multiplied by 1, the weight, is 16, which expreſſes it's power of percuſſion. Now, if a Fly of 3 pounds weight had been ſo attached to the 1 pound, that it's motion ſhould act with it, (without the 3 pounds falling at all, and that the motion of each ſhould be equal to the other,) for the reaſon above aſſigned, the velocity of the 4 pounds could be but as 2, for the ſquare of 2 is 4, which multiplied by 4 the quantity is 16 as before;—at 100 feet fall, the velocity of the ſingle pound would be as 10, the ſquare root of 100.—Thus do the proportions of acceleration continue the ſame, at the different diſtances of fall; but the actual proportion of motion will be leſs, as the Fly is heavier than the weight, which occaſions the power. If the Fly be ſo connected with the power, that it's velocity ſhall exceed that of the other, then will the retardation of the motion of the power exceed the preceding ſtatement in proportion to the ſquare of the exceſs.

Should the work to be done by a machine be perfectly regular and uniform, as well as the power applied, no advantage can be gained by the application of a Fly Wheel to it; for the expence of power in communicating motion to it, muſt be as great at leaſt as that which the wheel can return; but in proportion as the work or power is ſubject to variations, ſo ſhould the proportionate weight of the Fly, and it's velocity be greater or leſs, it will therefore be difficult to fix a general ſtandard; we may however conclude, that ſuch an auxiliary power ſhould never be greater than that which would juſt anſwer the purpoſe of regulating the machinery, as it muſt be attended with ſome friction, (however nicely ſuſpended;) which, as it depends much on the weight ſhould therefore be as little as needs be; beſides, that the heavier the Fly, the greater will be the difficulty of ſtopping the machine when neceſſary, and therefore the greater the power loſt in that operation.

SECTION X.

Of the Proportion of Reſiſtance proper to be applied to a Machine worked by the ſecond kind of Power, or Percuſſion.

IT is to be noted, that the preceding inveſtigation of the degree of motion neceſſary to be obtained in a machine, in order to work it to the greateſt advantage, applies only to ſuch as are impelled by a power of the third ſort, being connected with the machinery from the beginning of it's motion—but in ſuch as are acted on by percuſſion, another law obtains, which we will therefore proceed to enquire into, after recapitulating that we have endeavoured to ſhew, that the primary and obvious modes or qualities of independent matter in motion are three, viz.

Weight or quantity of matter,
Velocity, or quantity of motion,
Impetus or quantity of power.

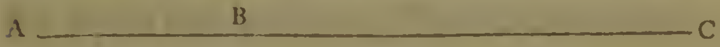
The comparative quantity of either two of theſe in two bodies being given, to find a third agreeably to the preceding doctrine.

Two bodies differing only in quantities of matter.	Their comparative Impetus will be in geometrical proportion to their weights or quantities of matter.
Bodies differing only in velocity or quantity of motion.	Their comparative Impetus will be as the ſquares of their comparative velocities in whatever numbers expreſſed.
Hence two bodies differing both in velocity and weight.	Their comparative Impetus will be as the ſquare of their velocity multiplied by their reſpective quantities of matter; 1lb. with the velocity of 10, is equal to 100lb. with the velocity 1lb. &c. &c.

Therefore two bodies whoſe impetus and velocities are known.	Their comparative quantities of matter are found, by dividing their impetus by the ſquare of their velocities.
Two bodies whoſe impetus and quantities of matter are known,	Their comparative velocities are found, by dividing the impetus by the quantities of matter reſpectively, and extracting the ſquare roots, which are the comparative velocities.
Hence if two bodies have different quantities of matter and equal impetus	Their comparative velocities will be as the ſquare root of each other's quantity of matter inverſely.
If two bodies have the ſame quantities of matter and different impetus	Their comparative velocities will be as the ſquare root of their impetus reſpectively.
If two bodies have different velocities and equal impetus	Their comparative quantities of matter will be as the ſquare of each others velocities.
If two bodies have the ſame velocities and different impetus	Their comparative quantities of matter muſt be as their impetus reſpectively

matter	velocity	inches
1 -	4 -	16
4 -	4 -	64
-	-	—
4 -	1 -	4
4 -	4 -	64

It is evident that agreeably to theſe data, if an independent body, having a certain degree of velocity overtake and ſtrike or impinge on another independent body, moving in the ſame direction, but with leſs degree of velocity, the impetus which they jointly poſſeſs after their union will be the ſame as that which they had in their ſeparate ſtates of motion; therefore to find the velocity with which they will procced conjointly—add their ſeparate impetus or percuſſions together, which divide by their joint weight, and from the product extract the ſquare root, it will be their joint velocity. Thus, a body of 6, with the velocity of 3, overtaking and ſtriking a body of 4 with the velocity of 1, they will proceed together with the velocity of 2, 402 nearly; but if the latter body be ſo attached to any matter in motion as to be hindered from moving on with the former, the impulſe it receives will be as the ſquare of the difference of velocities, multiplied by the quantity of matter ſo ſtriking; which therefore leads to a determination of the comparative degree of motion neceſſary to a wheel, or other firſt movement of machinery oppoſed perpendicularly to a current of water, wind, (or other matter in motion) for it to have the greateſt effect, or ſo as to do the moſt work. For ſuppoſe the line



A C of 30, to repreſent the velocity of a ſtream, of which A B of 10, is the velocity of a water wheel moved thereby, and laden with ſo much reſiſtence by work and friction as ſhall hinder it from partaking of more of the velocity of the ſtream; now, as the parts compoſing the body of the fluid are connected together, any check by the reſiſtence of the wheel on the parts immediately touching it, will occaſion thoſe behind to act on them, and ſo on to an indefinite diſtance; it will not therefore be neceſſary to take the quantity (or number of parts in the direction of the current,) into the account of their action, as it would be if the parts were unconnected, and only ſuch a quantity poſſeſſing a given ſpace, as that one ſhould have finiſhed it's effect before the other impinged.

Therefore, the ſquare of the difference of the velocities of the ſtream and wheel will repreſent the quantity of power which the latter will receive, or the work it will do, and the line A B the velocity or celerity with which it will do it; and it will be found, that this proportion is the moſt advantageous of any other, as is proved by trying the effect of that proportion, and varying it to a meaſure on either ſide, thus:

Square 20 is 400	which mult. by	vel. of work	10 is 4000
Now Sq. 21 is 441	do.	do.	9 is 3971
So ſq. 19 is 361	do.	do.	11 is 3969

And yet leſs would it be found by trying any further proportion; it therefore follows from this poſition, that as the ſquare of 30 is 900, (the whole action of the ſtream,) and the ſquare of 20 is 400, (the action of the ſtream on the wheel, when in motion as above) ſuch wheels ſhould be laden with $\frac{400}{900}$ or $\frac{4}{9}$ of the whole power of the ſtream, in order to do the moſt work to the beſt advantage.

SECTION XI.

Of the Action of Fluids in Motion on Plane Surfaces inclined to their Currents.

HAVING thus far treated of the *direct* action of fluids in motion, or bodies whoſe ſurfaces are oppoſed perpendicularly to their current, it remains only to enquire into their lateral effects on thoſe bodies whoſe ſurfaces decline from that poſition, and which are ſo confined as to yield only to the action, in a direction croſſing that of the ſtream; ſuch as the vanes of windmills, rudders of ſhips, &c.

Much has been written on the ſubject of theſe machines, to prove that there is one certain angle of poſition between the current of fluid, and the inclined ſurface which gives it the greateſt advantage; and this is generally agreed to be 54 deg. 44 min. —Why I cannot aſſent to this opinion will appear from the following inveſtigation, in which to proceed with perſpicuity, it is neceſſary to ſimplify the action and effect as much as poſſible.——Now it has been ſhewn, that the comparative powers of fluids (like thoſe of other bodies) in motion, are in their perpendicular effect, on equal ſurfaces of bodies at reſt, as the ſquare of their velocities; and ſhould the extent of their ſurfaces vary, it is evident that they muſt be as the ſquare of the fluid's velocity, multiplied by the ſurfaces of the ſuppoſed bodies;—which ſurfaces repreſent the quantity of fluid acting on them.—Thus, ſhould a body at reſt, preſent a plain ſurface of 2 ſquare feet (or 288 inches) to a current, it will be acted on by a force double of that by which a ſurface would which meaſured but 1 ſquare foot (or 144 inches) and as we are now only enquiring into the relative action of a current of the ſame velocity on different inclined planes this example is ſufficient to our purpoſe.---To underſtand the effect of a power applied to ſuch an inclined plane as is above mentioned, we may be aſſiſted by figure 3. plate 3.

Suppoſe the line A C, to repreſent the inclination of a plane, which with A B and B C form a triangular body ſo placed on friction wheels as to prevent it's moving but in a direction parallel to C B or B C; now, if a weight of 10 pounds be applied at A, and confined to a perpendicular action by a rod paſſing between rollers, &c. it's effect to move the triangle C B will be equal to 10 pounds, and it would ſuſtain a weight of 10 pounds attached to a line and carried over a pulley in the oppoſite direction, becauſe the baſe and ſide which form the triangle are equal; but if the line of inclination be carried to D, and A E and E D form the figure as before, ſo as to make A D double of A E, the weight 10 pounds would in that caſe ſupport 5 pounds only, and ſo on in the ſame proportion; that is in all caſes, as the line of the ſide of the triangle in the direction of the weight's action is to that which forms it's baſe, ſo will be the weight ſuſtained, (which repreſents the lateral action of the 10 pounds ſo confined as above,) to 10 pounds, which repreſents the direct action of the power:---it is therefore evident, that the leſs the angle made by the inclined plane with the line A B is, (being the direction of the perpendicular action of the weight,) the greater will be it's action on the plane in a lateral direction ; and as the action of the weight may repreſent the action of a column of fluid of any determinate volume, *(provided that fluid were not liable to be diverted from it's courſe by the re-action of the plane,)* we may fairly conclude from thence, that the leſs inclined any plane be when preſented to ſuch a volume, the greater power would the fluid have upon it, to put it in ſome degree of motion ; and that, in proportion as is the product of the ſide of the triangle divided by the baſe, (which repreſents the mechanical power of the inclined plane) multiplied by the baſe, (which alſo repreſents the direction of the volume of the fluid ;) but it is obvious, that if the inclination of a determined ſurface vary, the volume of the fluid it will embrace muſt alſo vary in the following proportion, according to the angle here laid down.

o. I	baſe of triangle	ſide of ditto.
5.44	1,000	9,950
11.33	2,000	9,797
17.27	3,000	9,540
23.35	4,000	9,165
30.00	5,000	8,660
36.52	6,000	8,000
45.00	7,071	7,071
44,26	7,000	7,140
53.08	8,000	6,000
64.09	9,000	4,360
90.00	10,00	0,000

For as the baſe is the diviſion of the ſide, to find the power of the plane, and alſo the multiplier of the product, (as repreſenting the volume of the fluid,) the power of action will ſtill be *as the ſide of the triangle*, and therefore greateſt, the leſs the angle which the inclined plane makes with the

direction

direction of the current, although not in the degree of the former caſe; but this concluſion (as was before obſerved) is under the premiſe, that " the fluid is not liable to be diverted in it's courſe by the re-action of the plane," as is the caſe with the weight of 10lb. acting as above:—what alteration of effect takes place by this re-action therefore remains to be conſidered, and I believe we ſhall find, that this important matter is ſubject to variation according to the circumſtances attending the fluid, which is the moving power; as firſt, in reſpect to the elaſticity of the fluid; for ſhould it have much of that property, and the angle of the inclined plane on which it ſtrikes be very acute, it will have a greater tendency to fly off with more force, and therefore leſs of it's power will be expended on the plane than if it had leſs of it. The ſame inclination therefore cannot be proper for air and water;—ſecondly, the more the acting power be perfect as a fluid, the more eaſily will it admit of an alteration in the direction of it's courſe by the re-action of the plane: and thirdly, if the current be more or leſs confined againſt the plane of an adjacent body, it will occaſion it to act with more or leſs of the diſadvantage that ariſes by the eaſy diverſion of it's courſe.

Theſe conſiderations, I fear, render it impoſſible to reduce the ſubject of the action of Fluids in general when in motion, on inclined planes, to any fixed or determinate rule: and were it otherwiſe, it is obvious, that the action of the Fluid as before defined, would apply only to the effort it would exert on the plane before it began to move, when no quantity of *time* can be taken into the calculation of it's effect upon it; but it is clear, (conſidering the inclined plane as a mechanical power oppoſed to the current,) that if the angle made by the plane be very acute with the direction of the current, it muſt not only be ſubject to the inconveniences we have before obſerved, and alſo leſſen exceedingly the volume of current it embraces, but muſt of it's remaining proportion loſe as much in time or ſpace as it gains in power.—On the other hand, if we apply it to the rudder of a ſhip, and it's angle to the current be brought near to a perpendicular, it will much impede the velocity of the ſhip; and if to the ſails of a windmill, it will divert the force of the wind applied to the ſails too much to that part of the axis which hinders their yielding to it's direct impulſe, and thereby occaſion the greater friction; in both caſes counteracting the deſired effect.

It may be alſo conſidered, that when the plane be laid, either very near or very remote from the direction of the current, the leaſt alteration in that direction may reduce it either to a parallel, or perpendicular line to it, and thereby deſtroy the whole intention.—Upon the whole of theſe conſiderations, therefore, I cannot think that any preciſe angle is determinable as poſſeſſing the greateſt advantage; but ſhould ſuppoſe that of 50 degrees from the current would be found as advantageous as any, making allowances for the various circumſtances attending the power as before mentioned: perhaps 51 deg. 44 min. may be as near, but the objection I make to it ariſes not only from the above reaſoning, but alſo from the appearance of falſe premiſes in determining on it.

If my mode of conſidering it be juſt, the variation of the angle can be conſidered only in the ſame view as any other mechanical contrivance preſented to a power, and in reſpect to the work to be accompliſhed by it, ſubject to the ſame rule as was laid down in the preceding ſection; that is, if the moving force and the power of the machinery be determined, it will do the ſame proportion of work to the beſt advantage, as a water-wheel (conſidered as a continued lever) was there ſaid to do; which is equal to four-ninths of what it would ſuſtain, (whether the lever contained in the wheel be long or ſhort,) and if the work and moving power be determined, the angle and extent of the inclined ſurface expoſed to the moving power muſt be regulated accordingly.

SUPPLEMENT.

MANY of the preceding thoughts were thrown together, with a view to improve in the beſt manner a mill worked by a ſtream of water, the power of which was in a great degree waſted; and having ſucceeded in it by a mode, which, though ſimple, appears to be new, I ſhall therefore attempt a deſcription of it, as it may not be altogether uſeleſs to ſuch as have any concern with machinery under the like circumſtances.

The ſtream alluded to is from a conſtant ſpring, affording about 400 cubic feet of water per diem, iſſuing from the ſide of a hill, which falls gradually through 48 perpendicular feet in the courſe of about 50 yards.—At this diſtance was a Corn Mill, having an overſhot wheel of 23 feet diameter;—from this ſpring therefore to the crown of the wheel is 25 feet fall, which ran to waſte, (except the action of the head of water in a reſervoir juſt above the wheel, of about 6 feet when full.)—Had not the Mill been already built, the erecting a wheel equal in diameter to ſo great a fall, and the inconvenience of connecting the machinery to it's ſhaft or axis at ſuch a height, would have been obſtacles too great to have permitted the application of the whole fall of the ſtream to one mill in the common way, and the reaſonings in the preceding part of this treatiſe determined the preference in my mind to the principle of overſhot wheels receiving water without percuſſion. I therefore conſidered that if an additional water-wheel, equal in diameter to the loſs of fall were erected on the ſide of the hill, wholly above the level of the old one, and both connected together, the ſame water might turn both, and perform (at leaſt) double the work which it did before: in conſequence I occaſioned a reſervoir to be made nearer the ſpring, from which pipes of ſix inches and an half bore were led down the hill to an upright tube which ſo delivered the water near the crown of the upper wheel as to turn it backwards.

The two water-wheels are connected by a chain paſſed round grooved wheels fixed to the arms of each, and as I conceived the firſt wheel went too faſt, when it gave a proper velocity to the ſtones, the upper chain-wheel was made of 15 feet diameter, and the under but of 10—which conſequently occaſioned the fall of water in the buckets, or revolution of the former, to be ſlower than the latter.

I had the ſatisfaction to find the ſucceſs exceed my expectation, as inſtead of double, the Mill will now do treble the work it would before with the ſame quantity of water, which is accounted for by the friction being but little encreaſed, though the power of the Mill be more than doubled. The figure will perhaps give a better idea of it than any deſcription by words only.

EXPLANATION.

A, the tube which brings the water from the reſervoir to

B, an upright tube, open on the top to ſuffer the water to riſe when ſuddenly ſtopt.

C, a horizontal tube which conveys the water to the upper wheel, and which has an aperture in the wider part of it's extremity, being opened and ſhut by a ſliding hatch connected to an upright lever which turns on a fulcrum at D, and which being joined to a horizontal one at E, and that leading to a ſmall lever at F, the hatch is opened and ſhut by a perſon within the Mill-houſe.

G and H, the upper and under wheel, connected by the two grooved chain-wheels I and K---the chain being 113 feet long and weighing one pound to a foot.

L, a double grooved roller in the interſection of the chain, to keep it's parts aſunder to prevent rubbing.

M, the trunk which conveys the water to the lower wheel after it has paſſed the upper one.

I ſhall conclude this treatiſe with the following deſcription of a Machine, which I have lately conſtructed, for a preſs in the proceſs of Cyder making, but which is applicable to many other ſimilar operations, and is more ſimple and leſs liable to injury or difficulty of repair than any other which has fallen under my obſervation; the power is alſo very great, being as 1 to 1136 nearly, and capable by a trifling addition of any other proportion.

A A Two pieces of timber 21 feet long, 12 by 6 inches, laid ſide by ſide at the diſtance of 12 inches, and ſecured in that ſituation by blocks placed between and bolts paſſing through them : this frame forms the bed of the machine.

B B Two uprights 12 feet long 6 by 8 inches morticed, upon them, and ſecured in their poſition by pins, and iron ſquares.

C C Two uprights 5 feet long 6 by 10 inches, morticed near the end of the under frame, and ſecured as before.

D a lever 17 feet long, 12 by 13 inches, turning on a large bolt which paſſes through the ſhort uprights—alſo through iron ſtraps, which ſecure them to the bed inſide, and a ſtirrup of iron which paſſes over the end of the lever, and which makes the turning point in the line of it's lower ſide and not through it's middle.

E a lever 20 feet long, 6 by 8 inches at it's largeſt part, and tapering towards the other end: this lever turns on a bolt in the uprights BB.

F 1, 2, 3, 4. Four pieces of oak (which I call needles 10 feet long) 4 by 2½ inches, mortiſed looſely into the upper lever, and hung thereto by bolts, ſo as to ſwing perpendicularly, and play in a long mortice or channel cut through the large lever to receive them. Theſe needles have inch-holes pretty cloſely bored through them (in a direction croſſing the machine,) from the lower ends, as far upwards as the great lever will reach, when it is as high as it can go.

G a bed to receive what is to be preſſed.

H a frame to ſupport a winch worked by a handle at I.—At the end of the ſmall lever two blocks or pulleys are fixed, one above and the other below it—a rope of about half an inch diameter is then faſtened to the cieling, (or continuation of the uprights of the winch frame if neceſſary,) at R; then rove through the upper block on the lever, from thence paſſed through a block at L, and then goes with four turns round the winch, from whence it is carried through the block under the lever and faſtens to the machine at M; by this means, if the winch be turned one way it raiſes the end of the ſmall lever, if the other depreſſes it.

To work the Machine.

If we ſuppoſe the great lever bearing on the matter to be preſſed, an iron pin muſt be put into one of the holes in the needles above the great lever, and when the ſmall lever is worked as far as it will go, either up or down, another bolt is to be put into the hole, which comes neareſt above the great lever on the other ſide of the uprights BB. and the winch then turned the contrary way, by which means the preſſing goes on whether the ſmall lever riſes or falls;—before the reſiſtance is very great, the needles fartheſt from the fulcrum of the ſmall lever are uſed, after that the neareſt are employed, which doubles the power of the machine.

In raiſing the great lever or lowering it to it's bearing, the needles moſt diſtant from the fulcrum of the ſmall lever are uſed *under* inſtead of *over* it.

As the rope is liable to ſtretch and get ſlack I paſs it, (after taking two turns on the winch, through a pulley, to which is ſuſpended a weight of half a hundred, and then take two turns more before it is carried through the other block, by which means the ſlack is conſtantly gathered in, and the weight *holds on* without increaſing the friction, as by hanging under the winch it counteracts the preſſure upwards on it's axis.

The direction of the preſſure of this Machine is not perpendicularly downwards, which is in ſome caſes an inconvenience; but by applying a ſharp or round billet between the lever, and the follower, which preſſes immediately on the ſubject of the work, it is in a great degree obviated—and an advantage of no ſmall import in addition to thoſe before mentioned, attending this Preſs above moſt others, eſpecially wooden ſcrews of the common kind, is the elaſticity of the levers which follow the preſſure after the winch is ſtopt from working; not to mention the *bevel* of a wooden (or other ſuch) ſcrew acting as a mechanical power to increaſe friction.

FINIS.

Alſo, publiſhed by the ſame Author, Price 2s. 6d.
Remarks on the comparative Advantages of Wheel Carriages of different Structure and Draught.

ERRATA.

In advertiſement line 4, for *io* read *to*.

Page 9 lines 7 and 8, for *Doc rine* read *Doctrine*.

13 line 13, for *Equally* read *Equably*.

34 line 7, for 18 read 16.

38 line 2, for *actual* read *natural*.

40 line 4, for $\frac{1}{7}$ read 7.

47 line 20, for *ages* read *pages*.

48 line 2, for *peritrechio* read *peritrochio*.

65 head of 3d column of figures, for *inches* read *Impetus*.

75 head of 1ſt column of figures, for 01 read *Angle*.

86 line 9, for *morti ed* read *morticed*.

Plate 2, Fig 1ſt, *E F* wanting.

Plate 1st

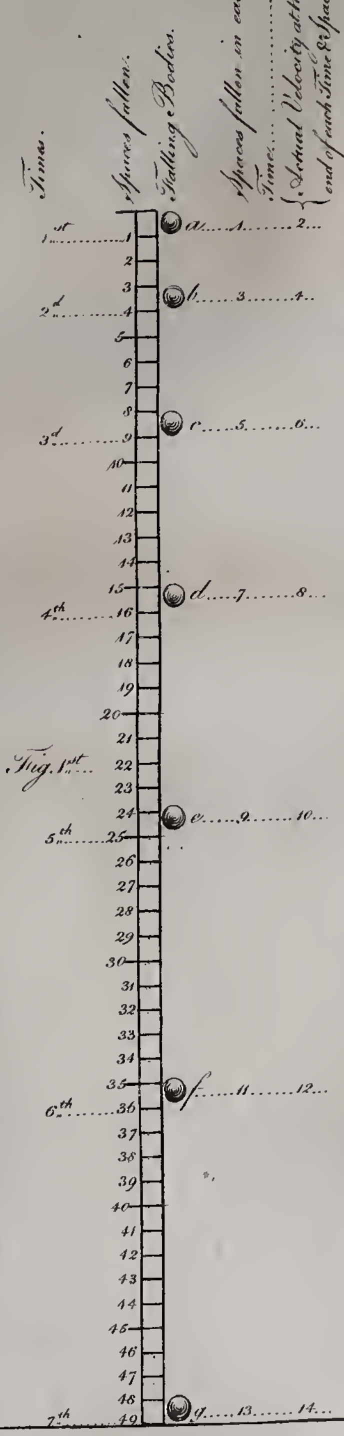


Fig. 1st

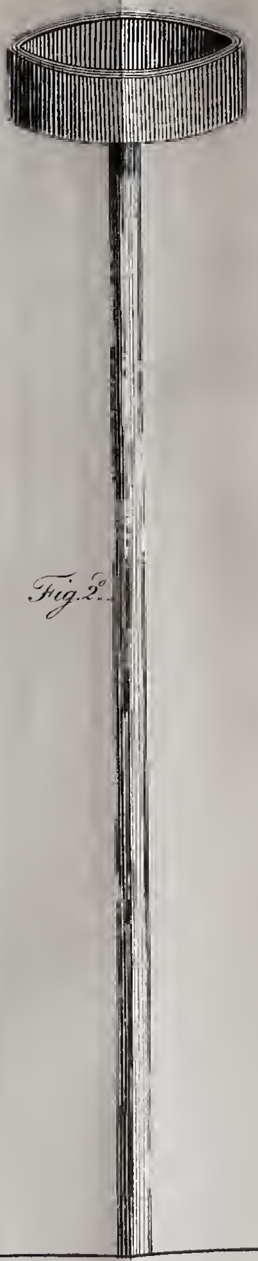


Fig. 2d

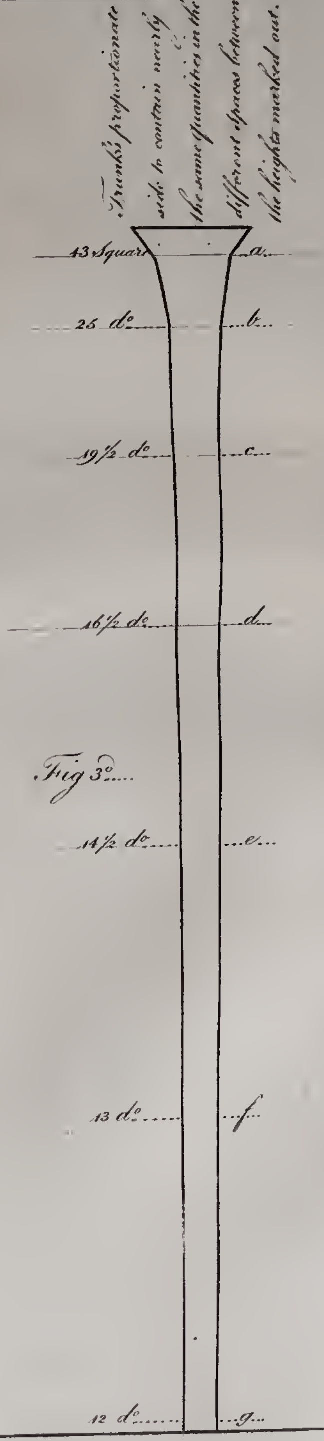

Trunk's proportionate side to contain nearly the same quantities in the different spaces between the heights marked out.
43 Square ... a
25 do ... b
19½ do ... c
16½ do ... d
14½ do ... e
13 do ... f
12 do ... g

Fig 3d

Plate. 2d

Fig. 1st

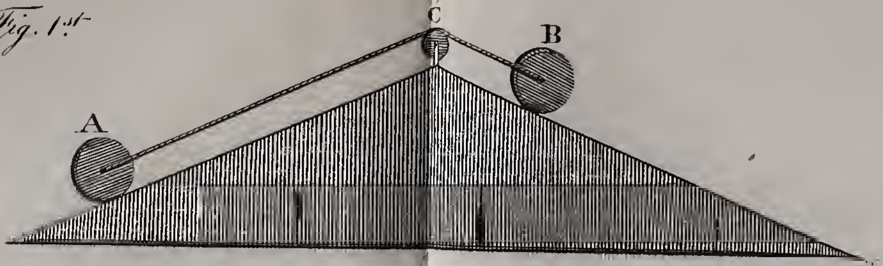


Fig. 2d

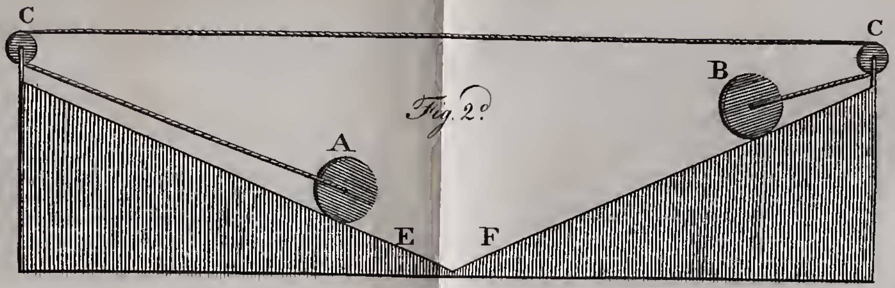


Fig. 3d

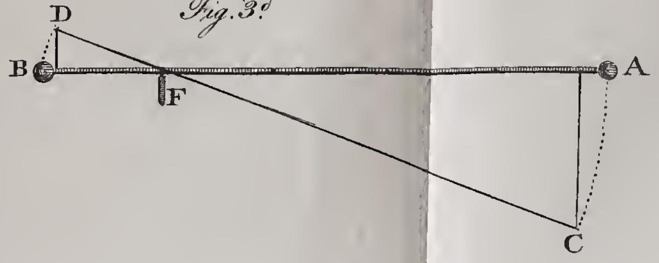
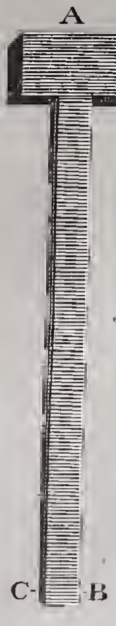


Fig. 4th



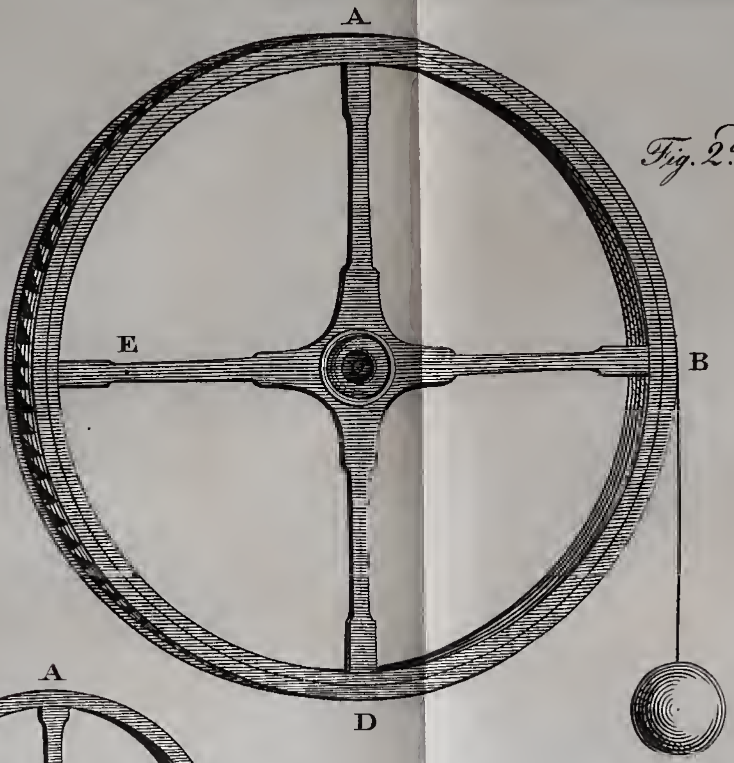


Fig. 2.d

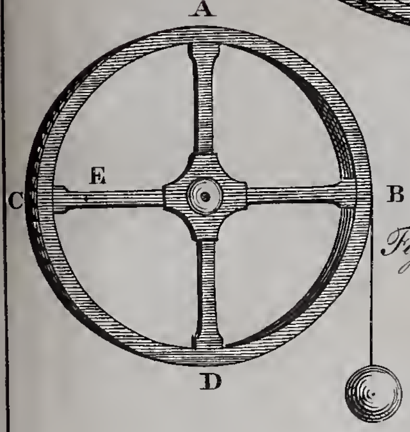


Fig. 1.st

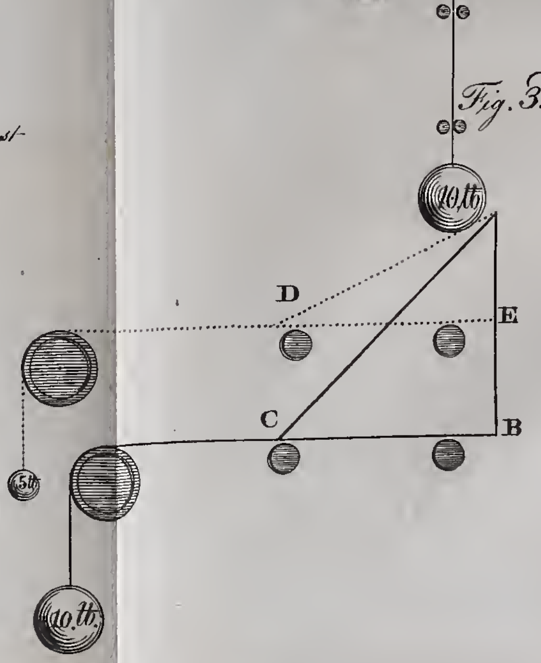


Fig. 3.d

Plate 4th

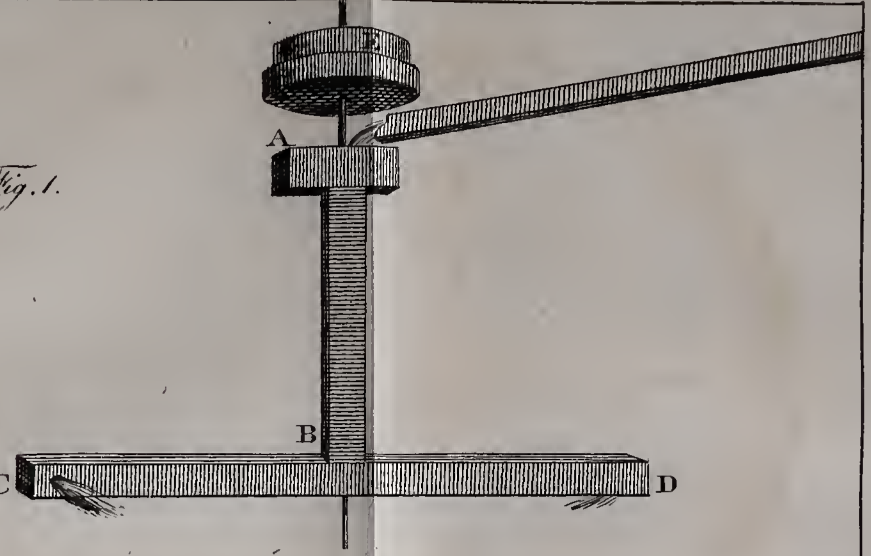


Fig. 1.

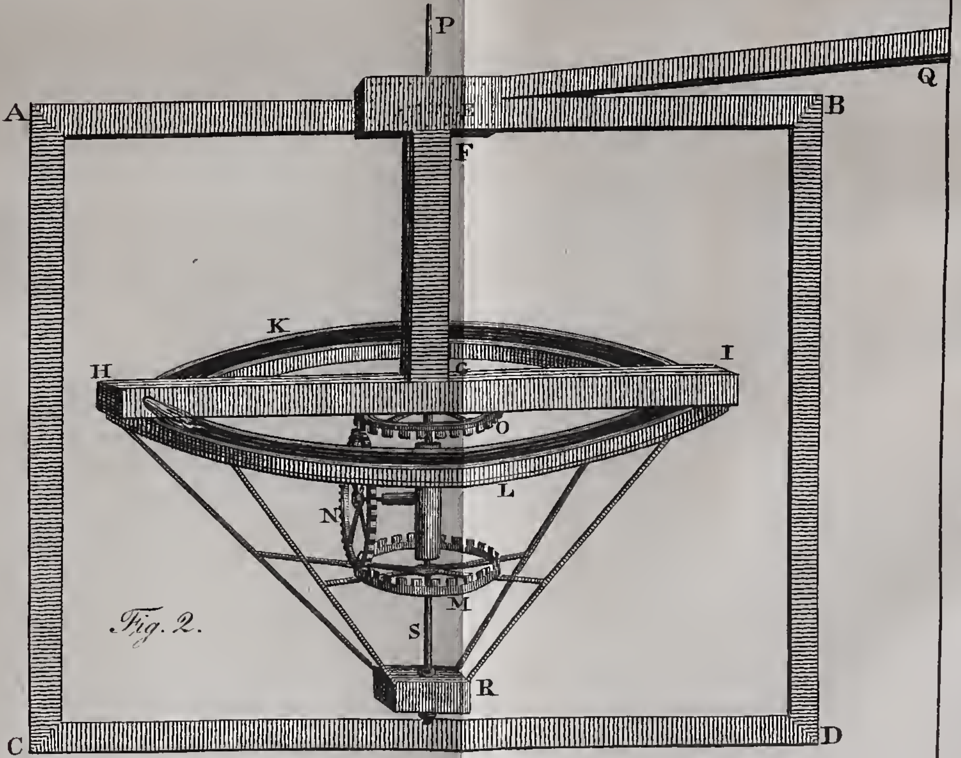
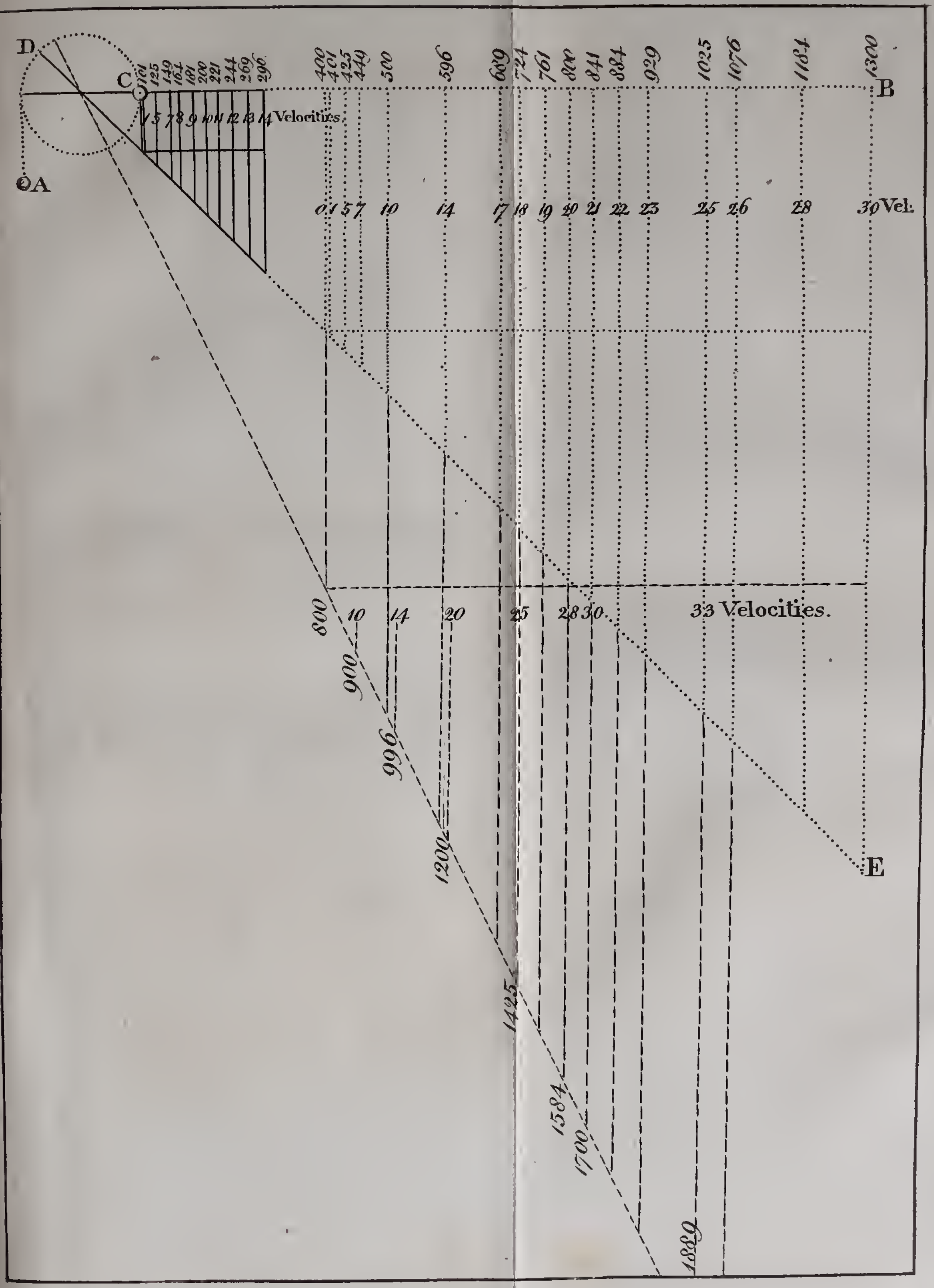


Fig. 2.

Plate 5th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C
A
B
E
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900
996
1200
1425
1584
1700
1880

Plate 6.

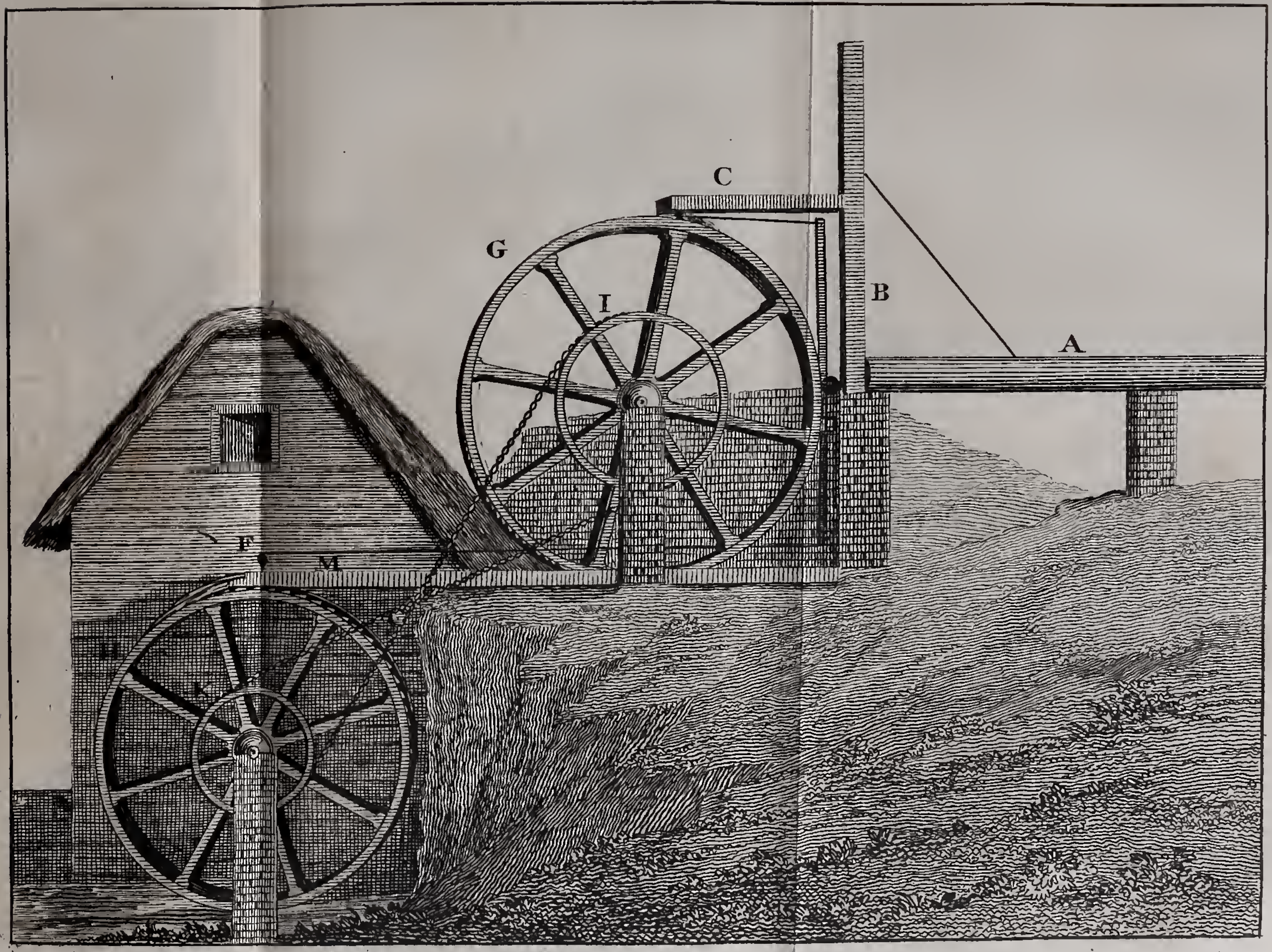

Plate. 7.